

Speed Limit on the Brane

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Relationship between the speed limit on the brane and in the bulk is discussed. We assume that the speed of light, similar to the 4-dimensional gravitational constant, is not a primary fundamental constant but depends on the gravitational potential of the brane. This opens the way to explain the hierarchy between the Plank and Higgs scales even within the simplest 5-dimensional model.

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According to Einstein's general principle of relativity the maximal velocity in our universe, c , is equal to the fundamental speed in Minkowski space-time. This equality is deeply embedded in Einstein's 4-dimensional field equations [1] and is confirmed experimentally [2]. Recent discussion about alternative claims that the constant entering space-time intervals, or the speed of gravity, is different from c can be found in [3].

Brane models usually also assumed that c is a universal constant. For attempts in the braneworld context, where the speed of gravity can be different from c , see [4].

We want to emphasize that the assumption that the maximal velocity in the bulk c_b coincides with the light speed on the brane c must not be taken for granted. The constant c_b that enters multi-dimensional Einstein equations corresponds to the velocity of massless particles in the empty bulk space-time. On the other hand c appears in the equations of 4-dimensional physics and possibly is not fundamental. For example, it can vary during the expansion of the universe. For the status of the present varying speed of light theories, see [5].

Note that the theory with

$$c \neq c_b \quad (1)$$

should operate with two metric tensors and thus will have two sets of "null cones", in the bulk and on the brane. This is the manifestation of violation of the bulk Lorentz invariance by the brane solution, in the sense that it will admit a preferred frame, the frame in which c and c_b are both isotropic. This possibility is strongly constrained for 4-dimensional physics [6], but can not be ruled out for multi-dimensional models.

Let us study some properties of the model with $c \neq c_b$ for the case of the simplest 5-dimensional brane *ansatz*

$$ds^2 = \phi^2(z) (g_{00}c_b^2 dt_b^2 - dl^2) - dz^2, \quad (2)$$

with the single space-like extra coordinate z . Here dl^2 is the metric of the 3-space on the brane and g_{00} describes the 4-dimensional gravitational potential, both are independent of z . The formula (2) contains the bulk quantities c_b and t_b and leads to the asymptotic 5-dimensional

Minkowski metric. If we take the brane to be located at $z = 0$ the well known example of gravitational warp factor in (2), which is responsible for the trapping of matter and Newtonian gravity on the brane, is

$$\phi \sim e^{-|z|/\epsilon}, \quad (3)$$

where the brane width ϵ is described by the value of the bulk cosmological constant [7].

Usually it is assumed that in the case which we call 4-dimensional vacuum

$$g_{00} \rightarrow 1, \quad \phi(z \rightarrow 0) \rightarrow 1. \quad (4)$$

However, because of trapping, the gravitational potential on a particle is not zero even in the case of 4-dimensional Minkowski metric and we can write instead

$$g_{00}\phi^2(z \rightarrow 0) \rightarrow U^2. \quad (5)$$

Here the dimensionless parameter U corresponds to the universal gravitational potential of the brane (that remains constant along it) and should be very small

$$U \ll 1. \quad (6)$$

Equating the kinetic and potential energies of a particle with the mass m in this gravitational field one can estimate the value of the escape velocity v from the brane

$$mc_b^2 U^2 \sim mv^2. \quad (7)$$

Then it is natural to identify the speed of light c with the escape velocity from the brane

$$c \equiv v = c_b U \ll c_b. \quad (8)$$

This means that c is not a universal constant, but is expressed through the gravitational potential of the brane.

From the assumption (8) follows that ideal clocks on the brane and in the bulk in general have different rates. From (8) and (2) for time intervals we get the relation

$$dt_b = U dt, \quad (9)$$

where t is time parameter on the brane. This is not surprising since the rate of a clock depends on the gravitational potential where it is placed. The brane time t

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is measured with the oscillations of classical or quantum objects using speed of light as an etalon. On the other hand the bulk time t_b is unknown quantity, since laws of physics can be much different there. Usual electromagnetic waves and gravitons are trapped on the brane and also we lack a theory that embodies multi-dimensional quantum physics.

Note that using (8), from (7) we obtain Einstein's famous formula for the rest energy of the body. So possibly the inertial mass m also has gravitational origin and is the measure of the gravitational interaction of the matter with the brane-universe. This is the another formulation of the Mach's principle and is opposite to the General Relativity interpretation of the equivalence principle [8].

One of the main ideas of brane models is that the 4-dimensional Planck scale m_{pl} is not a fundamental parameter, but expressed by the values of the extra volume V and the fundamental scale M (the scale at which the brane, viewed as a topological defect in higher-dimension space-time, exists) by the formula [9]

$$m_{pl}^2 = V \frac{M^{2+d}}{c^d}, \quad (10)$$

(we use the system of units where $\hbar = 1$) where d is the number of extra dimensions. This formula possibly explains famous hierarchy problem, why the gravitational interaction is much weaker than other interactions of particle physics.

It is known that the application of (10) to the 5-dimensional case ($d = 1$), when the Higgs scale $M \sim 1 \text{ TeV}$ is taken as fundamental, is problematic since we get a very large size of the extra dimension $\sim 10^{14} \text{ cm}$. However, this conclusion was obtained under the assumption that c is a universal constant.

Let us consider the 5-dimensional action integral for the *ansatz* (2)

$$S_g \sim \frac{M^3}{c_b^3} \int_{-\infty}^{\infty} dz e^{-|z/\epsilon|} \int dx^4 \sqrt{-g} R, \quad (11)$$

where R and g are respectively the scalar curvature and determinant in four dimensions and c_b is the limit speed in the bulk. On the other hand the action of matter fields on the brane, when $c \neq c_b$, will contain the constant c . Then the effective Planck scale in our model

$$\frac{m_{pl}^2}{c^2} = \frac{M^3 \epsilon}{c_b} = \frac{U^3 M^3 \epsilon}{c^3}, \quad (12)$$

is different from that what follows from (10). Obviously (12) can be generalized for any number of dimensions.

Now we have the extra free parameter U in the definition of effective Planck's scale (12). If we suppose, for example,

$$M \sim 1 \text{ TeV}, \quad U \sim 10^{-6}, \quad (13)$$

for the width of the brane, even for simplest 5-dimensional model, we find the realistic value

$$\epsilon \sim 10^{-3} \text{ mm}. \quad (14)$$

Note that deviations from the Newton law on short distances, which is predicted by the models with large extra dimensions, at present has been established experimentally down to the distances $\sim 10^{-1} \text{ mm}$ [10].

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